□ Class 11 – Mathematics

Chapter: Principle of Mathematical Induction

□ 1. Introduction

Mathematical Induction is a method of mathematical proof used to prove that a statement is true for all natural numbers.

□ 2. Need for Induction

Sometimes, verifying statements for a few values is not enough. We need a way to prove it holds true for all n□Nn \in \mathbb{N}n□N. That’s where induction is used.

□ 3. Steps of Mathematical Induction

To prove a statement P(n)P(n)P(n) is true for all n∈Nn \in \mathbb{N}n∈N:

Step 1: Base Case

Prove P(1)P(1)P(1) is true.
This step confirms the statement holds for the first natural number.

Step 2: Inductive Hypothesis

Assume P(k)P(k)P(k) is true for some arbitrary natural number kkk.
This assumption is not proof, it’s just a step.

Step 3: Inductive Step

Prove that P(k+1)P(k + 1)P(k+1) is true using the assumption that P(k)P(k)P(k) is true.
If successful, it shows P(n)P(n)P(n) is true for all n∈Nn \in \mathbb{N}n∈N.

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4. Example

Prove that:

1+2+3+…+n=n(n+1)21 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}1+2+3+…+n=2n(n+1)

□ Base Case:
For n=1n = 1n=1,
LHS = 1, RHS = 1(1+1)2=1\frac{1(1+1)}{2} = 121(1+1)=1 □ True

□ Inductive Hypothesis:
Assume true for n=kn = kn=k:

1+2+…+k=k(k+1)21 + 2 + \ldots + k = \frac{k(k+1)}{2}1+2+…+k=2k(k+1)

□ Inductive Step:
Add k+1k + 1k+1 to both sides:

1+2+…+k+(k+1)=k(k+1)2+(k+1)1 + 2 + \ldots + k + (k+1) = \frac{k(k+1)}{2} + (k+1)1+2+…+k+(k+1)=2k(k+1)+(k+1)

Simplify RHS:

=k(k+1)+2(k+1)2=(k+1)(k+2)2= \frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)(k+2)}{2}=2k(k+1)+2(k+1)=2(k+1)(k+2)

Which is the formula for P(k+1)P(k+1)P(k+1), hence proved.

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5. Applications

- Proving divisibility statements
- Sum of series
- Inequalities involving natural numbers

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6. Important Points

- Works only for natural numbers
- Always verify the base case
- Carefully use the inductive hypothesis in the inductive step
- Final conclusion: The statement holds for all $n \in \mathbb{N}$